Dynamic model of spherical perturbations in the Friedman universe. III. Automodel solutions.

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Abstract

A class of exact spherically symmetric perturbations of retarding automodel solutions linearized around Friedman background of Einstein equations for an ideal fluid with an arbitrary barotrope value is obtained and investigated.

1 Introduction

In the previous works of the authors [1,2] a class of exact retarding solutions for linear spherical perturbations of the Friedman universe with an ultrarelativistic equation of state of the ideal fluid filling it, corresponding to the central singular source presence and having the sight of polinoms by radial variable, was obtained. At that it was noted, by zero boarding conditions at the sound horizon for the C^1class metrics perturbations the energy dense perturbations have the first genus break at the sound horizon. In this paper we shall study the retarding solutions in detail by extending the investigations range to the equations of state of the fluid with an arbitrary barotrope coefficient κ.

So we shall investigate the retarding solutions of the evolutionary equation for the spherical perturbations [1] (84):

$$\ddot{\Psi}+\frac{2}{\eta}\dot{\Psi}-\frac{6(1+\kappa)}{(1+3\kappa)^2}\frac{\Psi}{\eta^2}-\kappa\Psi''=0. \tag{1}$$

with boarding conditions at the sound horizon corresponding to the zero values of the metrics perturbations and its first radial derivatives

$$\Sigma_0: \quad r=\sqrt{\kappa}\eta. \tag{2}$$

$$\Psi(r,\eta)|_{r\sqrt{\kappa}\eta}=\mu(\eta); \qquad \Psi'(r,\eta)|_{r\sqrt{\kappa}\eta}=0, \tag{3}$$

where $\mu(\eta)$ is the central source singular mass, so that the metrics component perturbation g_{44} is equal to

$$\delta g_{44}=a^2(\eta)\delta\nu, \quad \delta\nu=2\frac{\Phi(r,\eta)}{ar}=2\frac{\Psi(r,\eta)-\mu(\eta)}{ar}, \tag{4}$$

moreover the function $\Psi(r,\eta)$ corresponds to the nonsingular part of the potential:

$$\Psi(0,\eta)=0. \tag{5}$$

Temporal mass evolution is described by the equation

$$\ddot{\mu} + \frac{2}{\eta}\dot{\mu} - \frac{6(1+\kappa)}{(1+3\kappa)^2}\frac{\mu}{\eta^2} = 0, \tag{6}$$

which has the solution

$$\mu = \mu_+ \eta^{\frac{2}{1+3\kappa}} + \mu_- \eta^{-\frac{3(1+\kappa)}{1+3\kappa}}, \qquad (1+\kappa) \neq 0; \tag{7}$$

(for detail see the previous works [1,2]).
The potential function $\Psi(r,\eta)$ and the scalar $\mu(\eta)$ completely determine the energy density perturbations and the fluid velocity

$$\frac{\delta\varepsilon}{\varepsilon_0} = -\frac{1}{4\pi r a^3 \varepsilon_0}\left(3\frac{\dot{a}}{a}\dot{\Phi} - \Psi''\right), \tag{8}$$

$$(1+\kappa)v = -\frac{1}{4\pi r a^3 \varepsilon_0}\frac{\partial}{\partial r}\frac{\dot{\Phi}}{r}, \tag{9}$$

where $\varepsilon_0(\eta)$ is a non-perturbed energy density of the Friedman universe

$$\varepsilon_0 \sim \eta^{-\frac{6(1+\kappa)}{1+3\kappa}}; \quad a \sim \eta^{\frac{2}{1+3\kappa}}; \quad \varepsilon_0 a^3 \sim \eta^{-\frac{6\kappa}{1+3\kappa}}. \tag{10}$$

2 Automodel solutions

2.1 A general automodel solution

So we shall look for solutions of the evolutionary equations for the perturbations with zero boarding conditions at the zero sound horizon (2). Supposing that in (1):

$$\Psi(r,\eta) = \eta^\alpha G_\alpha(z), \tag{11}$$

where

$$z = \frac{r}{\sqrt{\kappa}\eta}, \tag{12}$$

we come to the *automodel solutions* class and get a common differential equation for the function $G_\alpha(z)$:

$$(1-z^2)\,G''_\alpha(z) + 2\alpha z\,G'_\alpha(z) + \left[\frac{6(1+\kappa)}{(1+3\kappa)^2} - \alpha(1+\alpha)\right]G(z)_\alpha = 0. \tag{13}$$

The potential function $\Psi(r,\eta)$ must be a combination of the private solutions

$$\Psi(r,\eta) = \sum_\alpha \eta^\alpha G_\alpha(z). \tag{14}$$

From (3) and (7) it follows that by $\alpha \neq 0$ this combination can have two members only

$$\Psi(r,\eta) = G_+(z)\eta^{\frac{2}{1+3\kappa}} + G_-(z)\eta^{-\frac{3(1+\kappa)}{1+3\kappa}}, \tag{15}$$

at that in consequence of the boarding conditions (3) the functions $G_\pm(z)$ must satisfy the following boarding conditions

$$G_\pm(1) = \mu_\pm; \qquad G'_\pm(1) = 0. \tag{16}$$

In particular from (3) and (8) it follows immediately that in the case $\alpha = 0$ the zero mass of the singular source corresponds to the pointed out class of solutions.

At the parameter α arbitrary values the general solution of the linear differential equation (13) is expressed by the Legendre functions, $P^\mu_\nu(z)$, and the adjoint Legendre functions Q^μ_ν[1]:

$$G(z) = \left(\frac{1-z}{1+z}\right)^{\frac{\alpha+1}{2}} \left[C_1 P^{\alpha+1}_{\frac{3(1-\kappa)}{2(1+\kappa)}}(z) + C_2 Q^{\alpha+1}_{\frac{3(1-\kappa)}{2(1+\kappa)}}(z)\right]. \tag{17}$$

2.2 An auto model solution with the particle-like source ($\mu \neq 0$)

The mentioned above is just for the formal general solution (17) at an arbitrary value of the parameter α. However, in our particular case (15) the values of the parameter α:

$$\alpha = \left(\frac{2}{1+3\kappa}, -\frac{3(1+\kappa)}{1+3\kappa}\right) \tag{18}$$

are the quadratic equation roots simultaneously

$$\frac{6(1+\kappa)}{(1+3\kappa)^2} - \alpha(1+\alpha) = 0. \tag{19}$$

Therefore in this case the equation (13) degenerates into a simpler one

$$(1-z^2)\, G''_\alpha(z) + 2\alpha z\, G'_\alpha(z) = 0, \tag{20}$$

However, integrating it we obtain the following

$$G'_\alpha(z) = C_1(1-z^2)^\alpha. \tag{21}$$

Comparing the second boarding condition (16) with the expression (21) we see that for the fulfillment of the zero conditions for the first derivative of the potential at the zero sound horizon it is necessary that $\alpha > 0$, that is The

[1] For example, see [7] and [8].

particle-like source mass is growing by the time and is equal to zero at the time moment $\eta = 0$; thus, =1 to provide a smooth sewing together of the solution with the Friedman one at the zero sound horizon it is necessary that

$$\mu_- = 0. \tag{22}$$

In the case of $\alpha > 0$in consequence of the relation (21) the second boarding condition (16) is fulfilled by fulfilling the first one automatically, thus, the condition $\alpha > 0$ fulfillment provides a smooth sewing together of the solution in the C^1 class at the sound horizon.
Now integrating the equation (21) formally and considering the conditions at the beginning of the coordinates (5), according to which

$$G(0) = 0, \tag{23}$$

we find its formal solution within the whole interval of the values $r = [0, +\infty)$:

$$G(\kappa, z) = C_1 \begin{cases} zF\left(\frac{1}{2}, -\frac{2}{1+3\kappa}, \frac{3}{2}, z^2\right), \quad (z \leqslant 1); \\ \dfrac{\sqrt{\pi}\Gamma\left(\frac{2}{1+3\kappa}+1\right)}{2\Gamma\left(\frac{2}{1+3\kappa}+\frac{3}{2}\right)} + \displaystyle\int_0^{\ln(z+\sqrt{z^2-1})} {}^{\frac{4}{1+3\kappa}+1} x dx, \quad (z > 1), \end{cases} \tag{24}$$

where $F(a, b, c, x)$ is a hyper-geometrical function (for example ,see [7]):

$$F(\alpha, \beta, \gamma, z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\alpha}, \tag{25}$$

$$\Re(\gamma) > \Re(\beta) > 0; \ |\mathrm{arg}(1-\mathrm{z})| < \pi.$$

At that the useful limitary relation is just [7]:

$$\lim_{z\to 1-0} F(\alpha, \beta, \gamma, z) = \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}, \quad \Re(\gamma-\alpha-\beta) > 0; \tag{26}$$

where $\Gamma(x)$ is Γ-function. In Fig. 1 the solutions (24) for values series of the barotrope coefficient are shown.

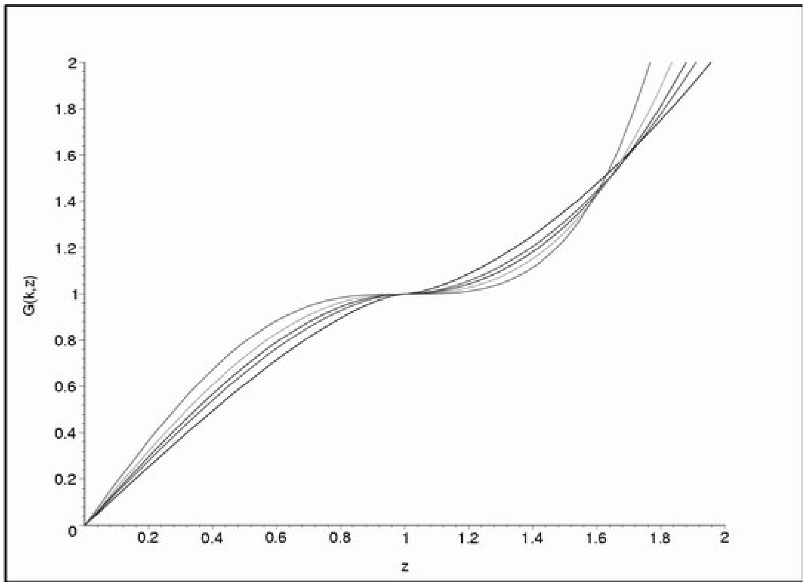

Fig. 1. Normalized function $G(k,z)$ calculated with the formula (24) at the value (27) of the constant C_1 and $\mu = 1$. In the left part of the figure from bottom up $\kappa = 0; \frac{1}{6}; \frac{1}{3}; \frac{2}{3}; 1$.

In terms of the obtained solution (24) it is easy to see $G(-z) = G(z)$, that is the function $\Psi(r,\eta)$ is an odd function of a radial variable really, it was shown in Ref. [2]. Considering the first boundary condition (16), we find the constant C_1:

$$C_1 = \mu_+ \frac{2\Gamma\left(\frac{2}{1+3\kappa} + \frac{3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{2}{1+3\kappa} + 1\right)} \tag{27}$$

and thus, finally we obtain the automodel C^1class solution corresponding to the zero boundary conditions at the sound horizon

$$\Psi(r,\eta,\kappa) = \mu_+ \eta^{\frac{2}{1+3\kappa}} \times$$

$$\begin{cases} \dfrac{2\Gamma\left(\frac{2}{1+3\kappa} + \frac{3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{2}{1+3\kappa} + 1\right)} \dfrac{r}{\sqrt{\kappa}\eta} F\left(\dfrac{1}{2}, -\dfrac{2}{1+3\kappa}, \dfrac{3}{2}, \dfrac{r^2}{\kappa\eta^2}\right), & (r \leqslant \sqrt{\kappa}\eta); \\ 1, & (r > \sqrt{\kappa}\eta), \end{cases} \tag{28}$$

Taking into consideration the relation (28) according to (4) we obtain the scalar function of the metrics perturbation $\delta\nu$:

$$\delta\nu(r,\eta) = -\frac{2\mu_+}{r}\times$$
$$\left[1-\frac{2\Gamma\left(\frac{2}{1+3\kappa}+\frac{3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{2}{1+3\kappa}+1\right)}\frac{r}{\sqrt{\kappa}\eta}F\left(\frac{1}{2},-\frac{2}{1+3\kappa},\frac{3}{2},\frac{r^2}{\kappa\eta^2}\right)\right]\chi(\sqrt{\kappa}\eta - r), \quad (29)$$

where $\chi(x)$ is the Heavyside function:

$$\chi(x) = \{0, \_x \leqslant 0; 1, \_x > 0. \quad (30)$$

In the private values series of the barotrope coefficient κ the obtained solution is expressed in elementary functions $\kappa = 1/3$ the ultra-relativistic fluid

$$G(1/3,z) = z - \frac{1}{3}z^3;$$

$$\Psi = \frac{3}{2}\mu_+\eta\left(z-\frac{1}{3}z^3\right); \quad \delta\nu = -\frac{2\mu_+}{r}\left[1-\frac{3}{2}\frac{r}{\sqrt{3}\eta}+\frac{1}{2}\left(\frac{r}{\sqrt{3}\eta}\right)^3\right]; \quad (31)$$

Being auto-model the solution (31) coincides with the general solution in the form of the degree series, earlier obtained in the works [3,4,5,6], it confirms the correctness of the proved in [2] theorem about the retarding solution uniqueness in the case of the ultra-relativistic state equation once more. $\kappa = 1$: fluid with extremely stiff state equation

$$G(1,z) = \frac{1}{2}z\sqrt{1-z^2}+\frac{1}{2}\arcsin z;$$
$$\Psi = \frac{2\mu_+}{\pi}\sqrt{\eta}\left(\frac{r}{\eta}\sqrt{1-\frac{r^2}{\eta^2}}+2\arcsin\frac{r}{\eta}\right);$$
$$\delta\nu = -\frac{2\mu_+}{r}\left[1-\frac{2}{\pi}\left(\frac{r}{\eta}\sqrt{1-\frac{r^2}{\eta^2}}+\arcsin\frac{r}{\eta}\right)\right]. \quad (32)$$

Note, the first derivative conversion by a radial variable at the zero sound horizon in the obtained solutions at $\alpha > 0$ guaranties the relation (21).

2.3 A solution without a particle-like source

Now supposing that in (11) $\alpha = 0$, we get the following equation instead of (13)

$$(1-z^2)G''(z)+\frac{6(1+\kappa)}{(1+3\kappa)^2}G(z) = 0. \quad (33)$$

The general solution of this equation is a linear combination of the hypergeometrical functions

$$G(\kappa, z) = (1 - z^2) \times$$

$$\left[zC_1 F\left(\frac{3\kappa}{1+3\kappa}, \frac{5+9\kappa}{2+6\kappa}, \frac{3}{2}, z^2\right) + C_2 F\left(\frac{2+3\kappa}{1+3\kappa}, \frac{3\kappa-1}{2+6\kappa}, \frac{1}{2}, z^2\right)\right]. \quad (34)$$

If the geometrical functions

$$F\left(\frac{3\kappa}{1+3\kappa}, \frac{5+9\kappa}{2+6\kappa}, \frac{3}{2}, z^2\right) \quad and \quad F\left(\frac{2+3\kappa}{1+3\kappa}, \frac{3\kappa-1}{2+6\kappa}, \frac{1}{2}, z^2\right)$$

remained final at the sound horizon $z = 1$, the general solution (33) would automatically turn into zero at the sound horizon, then the zero boundary conditions at the sound horizon (16) would be in principle satisfied by the choice of constants in the general solution (33). However the pointed out hyper-geometrical functions have some peculiarities at the sound horizon.
Really, as it is easy to see, the parameters α, β, γ of the hyper-geometrical functions of this linear combination satisfy the condition (See the hyper-geometrical function definition (26).)

$$\alpha + \beta - \gamma = 1, \quad (35)$$

At that for the first member of this combination the additional condition is fulfilled

$$\gamma_1 - \beta_1 = -\frac{1}{1+3\kappa}, \quad (36)$$

at the same time for the second one it is

$$\gamma_2 - \beta_2 = +\frac{1}{1+3\kappa}. \quad (37)$$

In this case it is necessary to use the functional relation for the hyper-geometrical function [7], which conformably to (35) is given with the formula

$$F(\alpha, \beta, \alpha+\beta-1, z) = \frac{1}{(1-z)} F(\alpha-1, \beta-1, \alpha+\beta-1)N \quad (38)$$

Using this relation in the formula (34) let us rewrite the general solution in the form

$$G(\kappa, z) = (1+z) \left[zC_1 F\left(-\frac{1}{1+3\kappa}, \frac{3(1+\kappa)}{2(1+3\kappa)}, \frac{1}{2}, z^2\right) + \right.$$
$$\left. C_2 F\left(\frac{1}{1+3\kappa}, \frac{3(1+\kappa)}{2(1+3\kappa)}, -\frac{1}{2}, z^2\right)\right]. \quad (39)$$

Calculating the hyper-geometrical functions values in the right part of Eq. (39) at the sound horizon we see these functions have peculiarities at the sound horizon. Therefore the auto-model solution satisfying the zero boundary conditions at the sound horizon in the case under investigation is the trivial solution $C_1 = C_2 = 0$ only. Summarizing this subsection let us formulate the theorem
Theorem. *There are no retarding spherically-symmetric auto-model solutions of the equation* (1) *satisfying the zero boundary conditions* (16) *at the sound horizon without the central particle-like source* ($\mu = 0$).

The proved theorem is analogues to the theorem about the analytical solution of the Laplace equation in the spherical symmetry case.

3 Research of the auto-model solutions

3.1 Derivatives of the potential functions

Let us come over to the analyses of the obtained auto-model solutions in the particle-like source case. For that let us use the expression (8) of the relative energy density of perturbation and the expression (9) of the radial medium velocity in the perturbation. At that we shall need the expressions for the first and second derivatives of the potential functions. Considering the definitions (11) and (12) and the relations (20) and (27) we shall get expressions for the first and second radial derivatives of the potential functions $\Psi(r, \eta)$

$$\Psi'_r(r,\eta) = \mu_+ \eta^{\frac{1-3\kappa}{1+3\kappa}} \frac{2\Gamma\left(\frac{2}{1+3\kappa} + \frac{3}{2}\right)}{\sqrt{\kappa\pi}\Gamma\left(\frac{2}{1+3\kappa} + 1\right)} (1 - z^2)^{\frac{2}{1+3\kappa}}; \qquad (40)$$

$$\Psi''_{rr}(r,\eta) = -\frac{8\mu_+ \eta^{-\frac{6\kappa}{1+3\kappa}}}{\sqrt{\pi}} \frac{\Gamma\left(\frac{2}{1+3\kappa} + \frac{3}{2}\right)}{\kappa(1+3\kappa)\Gamma\left(\frac{2}{1+3\kappa} + 1\right)} (1 - z^2)^{\frac{1-3\kappa}{1+3\kappa}}, \qquad (41)$$

and for the temporal derivative of the function $\Phi(r, \eta)$ also

$$\dot{\Phi}(r,\eta) = \eta^{\frac{1-3\kappa}{1+3\kappa}} \left[2\frac{G(k,z) - \mu_+}{1+3\kappa} - \frac{2\mu_+ \Gamma\left(\frac{2}{1+3\kappa} + \frac{3}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{2}{1+3\kappa} + 1\right)} z (1 - z^2)^{\frac{2}{1+3\kappa}} \right], \qquad (42)$$

here the function $G(\kappa, z)$ is determined by the relation (24) with the constant C_1 from (27).

From the given expressions it follows: by $\kappa > -1/3$ the first radial and temporal derivatives of the potential functions $\Psi(r, \eta)$ and $\Phi(r, \eta)$ turn into zero at the sound horizon

$$\left.\frac{\partial}{\partial r}\Psi(r,\eta)\right|_{r=\sqrt{k}\eta} = \left.\frac{\partial}{\partial r}\Phi(r,\eta)\right|_{r=\sqrt{k}\eta} = 0; \quad (1 + 3\kappa \geqslant 0); \qquad (43)$$

$$\left.\frac{\partial}{\partial\eta}\Psi(r,\eta)\right|_{r=\sqrt{k}\eta} = \left.\frac{\partial}{\partial\eta}\Phi(r,\eta)\right|_{r=\sqrt{k}\eta} = 0; \quad (1+3\kappa \geqslant 0). \tag{44}$$

At $\kappa < 1/3$ the second radial derivatives of the potential functions turn into zero at the sound horizon

$$\left.\frac{\partial^2}{\partial r^2}\Psi(r,\eta)\right|_{r=\sqrt{k}\eta} = \left.\frac{\partial^2}{\partial r^2}\Phi(r,\eta)\right|_{r=\sqrt{k}\eta} = 0; \quad (1-3\kappa \geqslant 0). \tag{45}$$

By $\kappa = 1/3$ the second radial derivatives of the potential functions have a break of the first genus at the sound horizon. By $\kappa > 1/3$ the second radial derivatives of the potential functions have a break of the second genus at the sound horizon.

3.2 Evolution of the energy density distribution in the spherical perturbation

Calculating the relative density of the perturbation energy by the formula (8) considering the relations (10) and (40),(41) we get finally

$$\frac{\delta\varepsilon}{\varepsilon_0} = -\frac{1}{z\eta\pi\sqrt{\kappa}(1+3\kappa)}\times$$

$$\left[3\frac{G(k,z)-\mu_+}{1+3\kappa} - 3\frac{\mu_+\Gamma\left(\frac{7+9\kappa}{2(1+3\kappa)}\right)}{\sqrt{\pi}\Gamma\left(\frac{3(1+\kappa)}{1+3\kappa}+1\right)}z(1-z^2)^{\frac{2}{1+3\kappa}} + \right.$$

$$\left. + \frac{2\mu_+\Gamma\left(\frac{7+9\kappa}{2(1+3\kappa)}\right)}{\sqrt{\pi}\Gamma\left(\frac{3(1+\kappa)}{1+3\kappa}+1\right)}(1-z^2)^{\frac{1-3\kappa}{1+3\kappa}}\right]\chi(1-z) = \frac{\mu_+}{\eta}\Delta(z)\chi(1-z), \tag{46}$$

where the reduced relative density of the perturbation energy $\Delta(z)$ is introduced. It can be strictly shown that $\Delta(z) \geqslant 0$.

From this expression it is seen that by the time the profile form of the perturbation energy density does not change relatively the dimensionless radial variable $z = r/\sqrt{k}\eta$ and the relative density of the perturbation energy decreases in inverse proportion to the temporal variable η (Fig.2).

At that in the terms of the common radial variable r the energy density perturbation profile is deformed. For example in Fig.3 the evolution of the relative energy density perturbation profile at the barotrope index $\kappa = 1/6$ is shown. =3 Further on from the formula (46) it is immediately seen that at $\kappa < 1/3$ the energy density perturbation at the sound horizon vanishes, at $\kappa = 1/3$ it has a final jump at the sound horizon, and at $\kappa > 1/3$ it has an infinite jump, it absolutely corresponds to the behavior of the second radial derivatives of the potential functions.

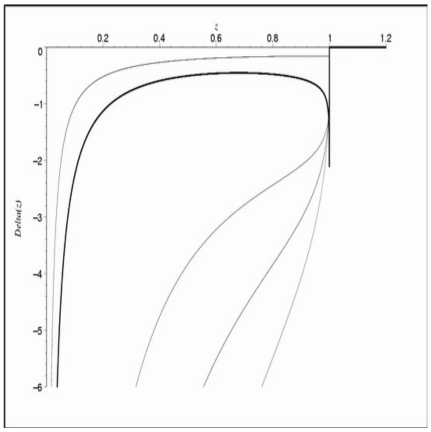

Fig. 2. It is the evolution of the reduced relative density of the perturbation energy Δ(z) calculated by the formula (45) as the function z. From bottom up in fine line $\kappa = 1/6; 1/5; 1/4; 1/3$; the heavy line corresponds to $\kappa = 1/2$.

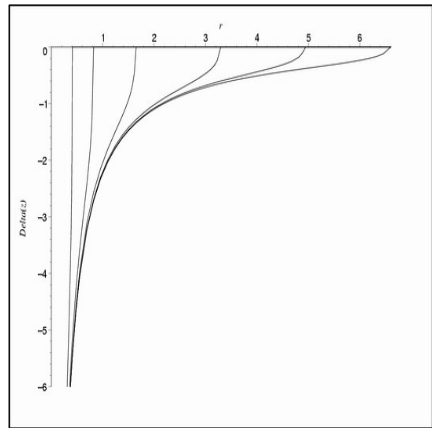

Fig. 3. It is the evolution of the reduced relative density of the energy Δ(z) calculated by the formula (45) as the function r at $\kappa = 1/6$. From the left to the right $\eta = 1; 2; 4; 8; 12; 16$.

Let us find out the physical sense of the obtained solution. The energy perturbation corresponding to the nonsingular part of the potential function is described by the formula

$$\delta E = 4\pi a^3 \int_0^{\sqrt{\kappa}\eta} \delta\varepsilon r^2 dr.$$

Hence considering (10) we get

$$\delta E = 4\pi\eta^{-\frac{6\kappa}{1+3\kappa}} \int_0^{\sqrt{\kappa}\eta} \frac{\delta\varepsilon}{\varepsilon_0} r^2 dr.$$

Coming over to the dimensionless variable z in the integral by the formula

$$r = \sqrt{\kappa}\eta z,$$

we bring it to the form

$$\delta E = 4\pi\mu_+\eta^{\frac{2}{1+3\kappa}}\kappa^{3/2} \int_0^1 \Delta(z)dz \sim -m(\eta), \tag{47}$$

where (see (7)):

$$m(\eta) = \mu_+\eta^{\frac{2}{1+3\kappa}} \tag{48}$$

is the mass of the central singular particle-like source. Thus, the full energy in the included in the nonsingular mode of the perturbation is negative and proportional to the mass of the particle-like source.

3.3 Evolution of the fluid radial velocity in the spherical perturbation

Fulfilling the analogues calculations we get the expression for the radial velocity from (9)

$$v = \frac{3}{8\pi\eta^2\kappa^{3/2}(1+3\kappa)}\left[\frac{G(k,z)-\mu_+}{z^3} - \frac{\mu_+\Gamma\left(\frac{7+9\kappa}{2(1+3\kappa)}\right)}{\sqrt{\pi}\Gamma\left(\frac{3(1+\kappa)}{1+3\kappa}+1\right)}\frac{(1-z^2)^{\frac{2}{1+3\kappa}}}{z}\right.$$

$$\left.-4\frac{\mu_+\Gamma\left(\frac{7+9\kappa}{2(1+3\kappa)}\right)}{\sqrt{\pi}\Gamma\left(\frac{3(1+\kappa)}{1+3\kappa}+1\right)}(1-z^2)^{\frac{1-3\kappa}{1+3\kappa}}\right] = \frac{1}{\eta^2}\Upsilon(z). \qquad (49)$$

From this expression it is also seen that the radial velocity is negative and its profile remains constant within the scale z, and the absolute value of the velocity drops in inverse proportion to η^2.

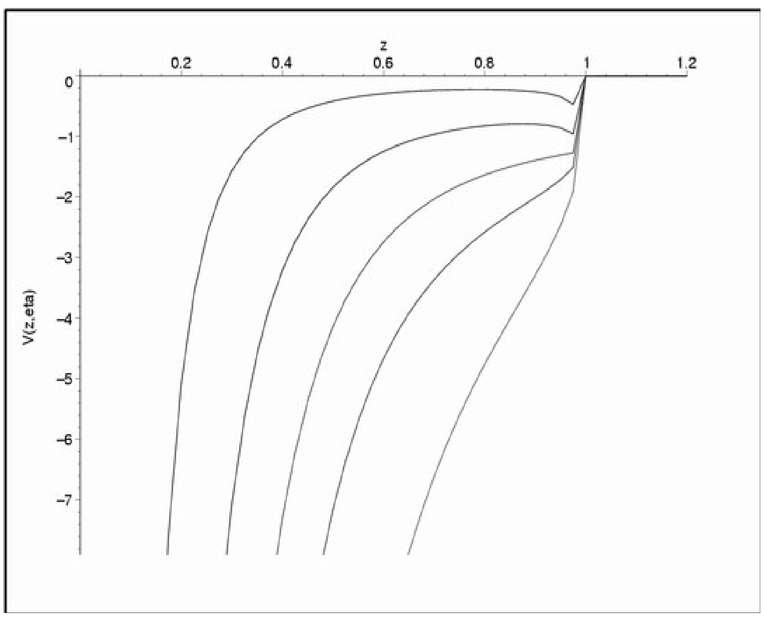

Fig. 4. Dependence of the profile of the perturbation radial velocity $\Upsilon(z)$ on the barotrop coefficient at$\kappa = 1/6$. From bottom up $\kappa = 1/6; 1/4; 1/3; 1/2; 1$

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